**Group Theory**

* **Groupoid or Binary Algebra Definition**
  + A non-empty set G equipped with one binary operation is called groupoid
    - i.e. G is a Groupoid is closed for \* .

It is denated by (G,\*).

For Example :- (N,+), (Z, -), (Q, x) etc.

Note :- Groupoid is also called Quasi Group.

* **Semi Group :- Definition** 
  + An Algebraic Structure (G, \*) is called a semi Group if the binary operation \* satisfy associative Property

i.e. :- [G] (a \* b) \* c = a \* (b \* c), a ∈ G

Ex.1

The algebraic structures (N,+), (Z,+), (Z, x), (Q, x) are semi groups but the structure (Z, - ) is not so because subtraction, (-) is associative.

Ex. 2

The structures (p(s), u) and (P(s), ∩) where P(s) is the power set of a set S are Semi Groups as both the Operations Union (U) an intersection (∩) are associative.

**Monoid : Definition**

* + A Semi Group is called monoid if there exist an identity element ‘e’ in G such that
    - [G2] e \* a = a \* e = a, a **∈ G**
* Ex.1

The Semi Group (N, x) is Monoid because 1 is the identity for the multiplication. But the Semi Group (N, +) is not because 0 is the identity for addition is not in N.

* Ex. 2

The Semi Group (P(s), U) and (P(s), ∩) are monoid because  and S are the identity respectively for union (U) and (∩) in P(s).

**Group: Definition**

* + An algebraic structure of set G and a binary operation \* defined in G i.e. (G, \*) is called a group if \* satisfies the following postulate :

**[G1] Closure :**

**[G2] Associativity :**

* + - The composition \* is associative in G

i.e.

**[G3] Existence of Identity :**

* + - There exist an identity element e in G such tha

**[G4] Existence of Inverse :**

* + Each element of G is invertible, i.e., for every , there exist in G such that  
     Identity)
  + Thus group (G, \*) is a monoid in which each of its element is ...

**Abelian group or Commutative group : Definition**

* + - * A group is said to be abelian or commutative if \* is commutative also.  
        A group is an abelian group, if

**[G4] Commutativity:**

**G1. Associativity**

**G2. Existence of Identity**

**G3. Existence of Inverse**

**G4. Commutativity**

**Finite and Infinite groups :**

* + - **A group (G, \*) is said to be finite if its underlying set G is finite set and a group which is not finite is called an infinite group.**

**Order of a Group : Definition**

* + - **The number of elements in a finite group is called the order of the group.**
    - **It is denoted by O(G).**
    - **If (G,\*) is in infinite group, then it is said to be of infinite order.**

**Example – 1**

* **Show that the set of integers forms an abelian group under addition.**

**Solution :**

* Let
* **: Closure :**
* let   
   Closure law is satisfied.
* **: Associative :**
* let
* Associative law is satisfied.
* **: Identity** : let and be the identity
* **G4​: Inverse :**
* is the inverse of
* **: Commutative** :
* let
* is an abelian group under addition.

**Example – 2**  
 **G is the set of rationals except -1. Binary operation is defined by  
. Show that it is a group.**

**Solution:**

: Let   
Closure is satisfied.

: Let

L.H.S.: